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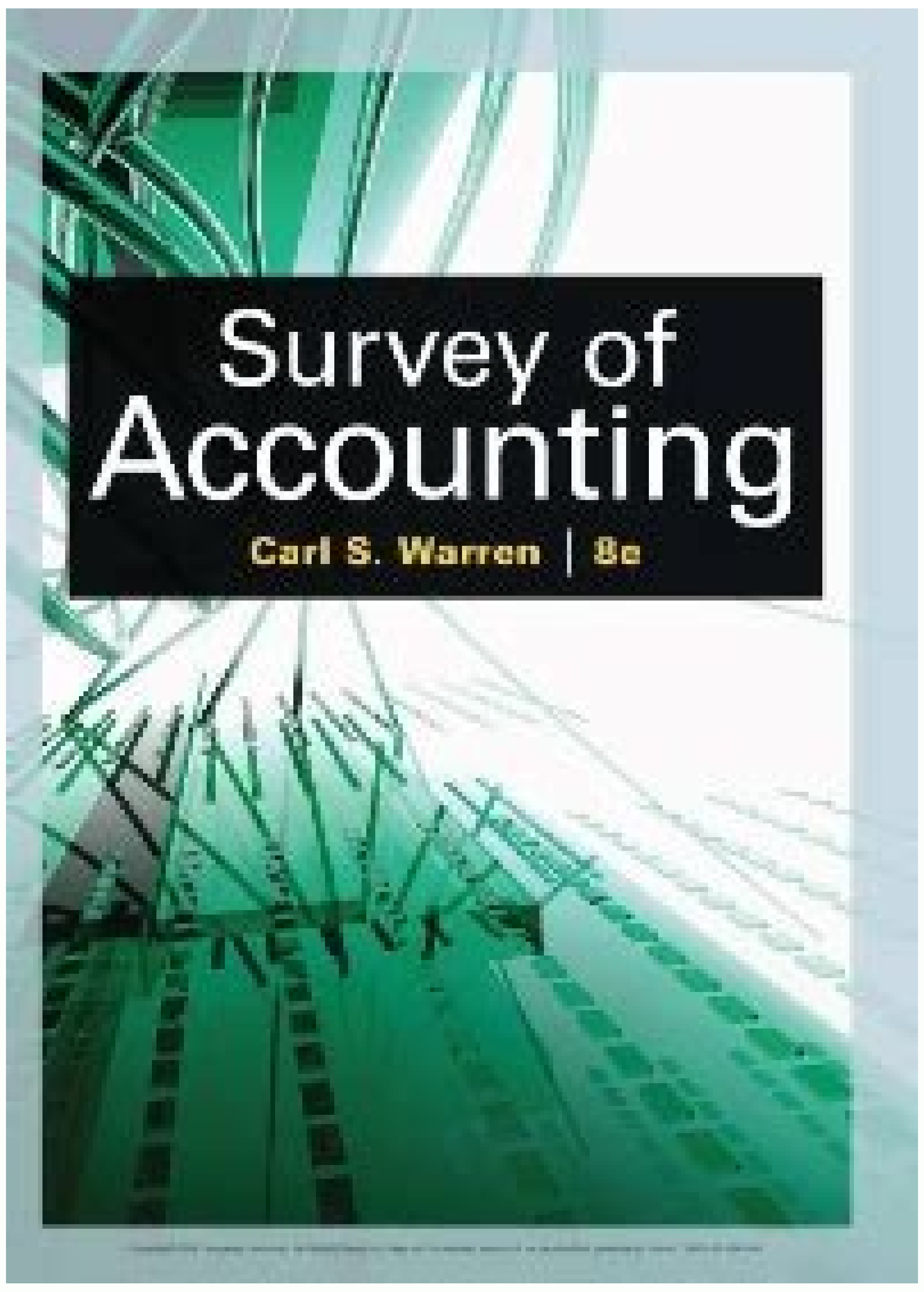
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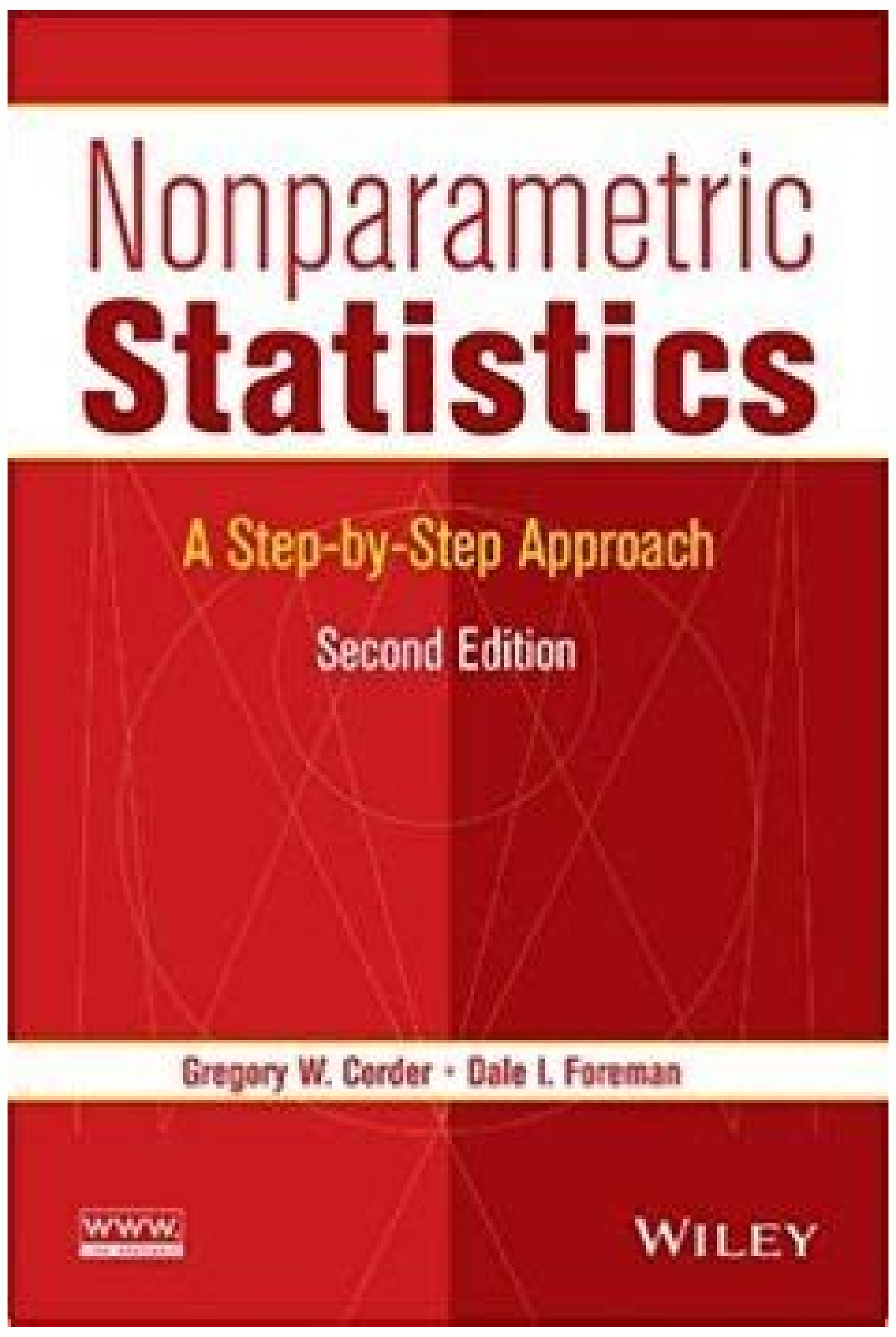
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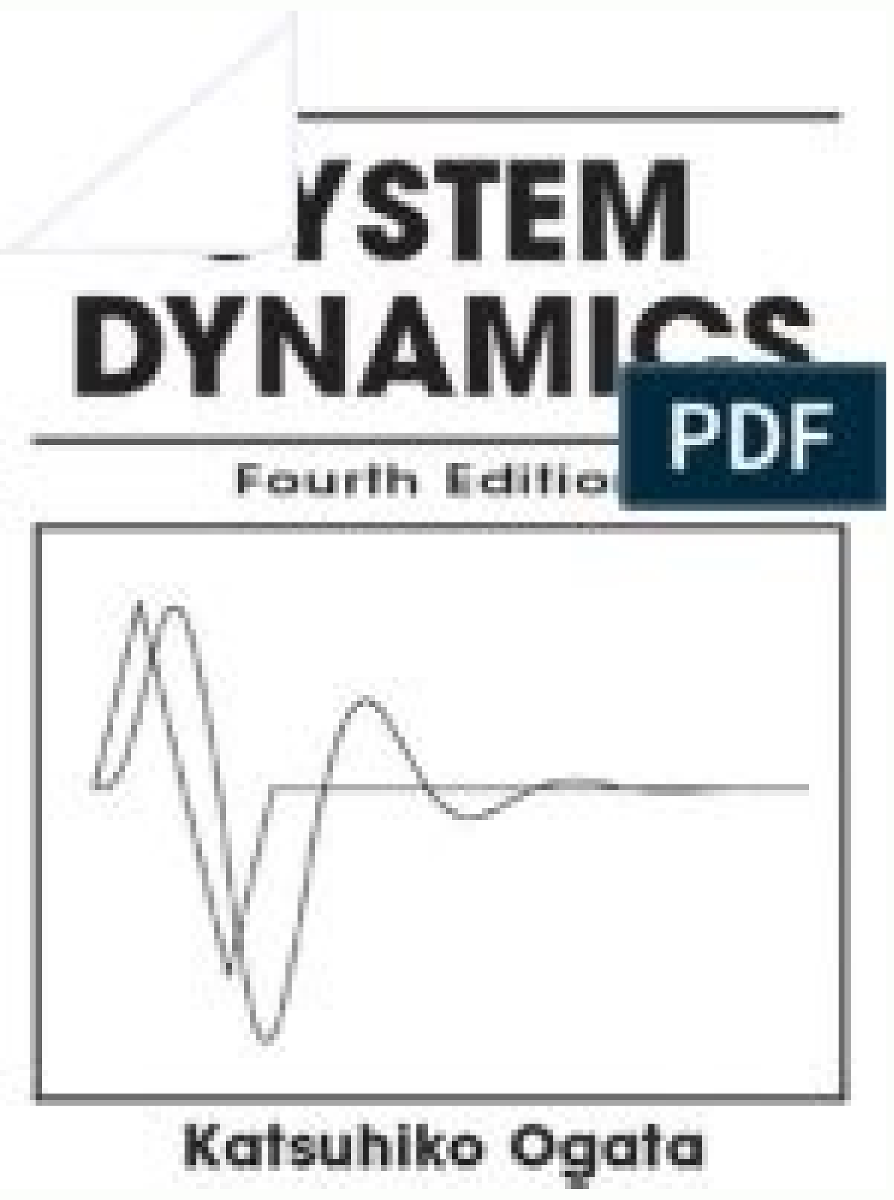
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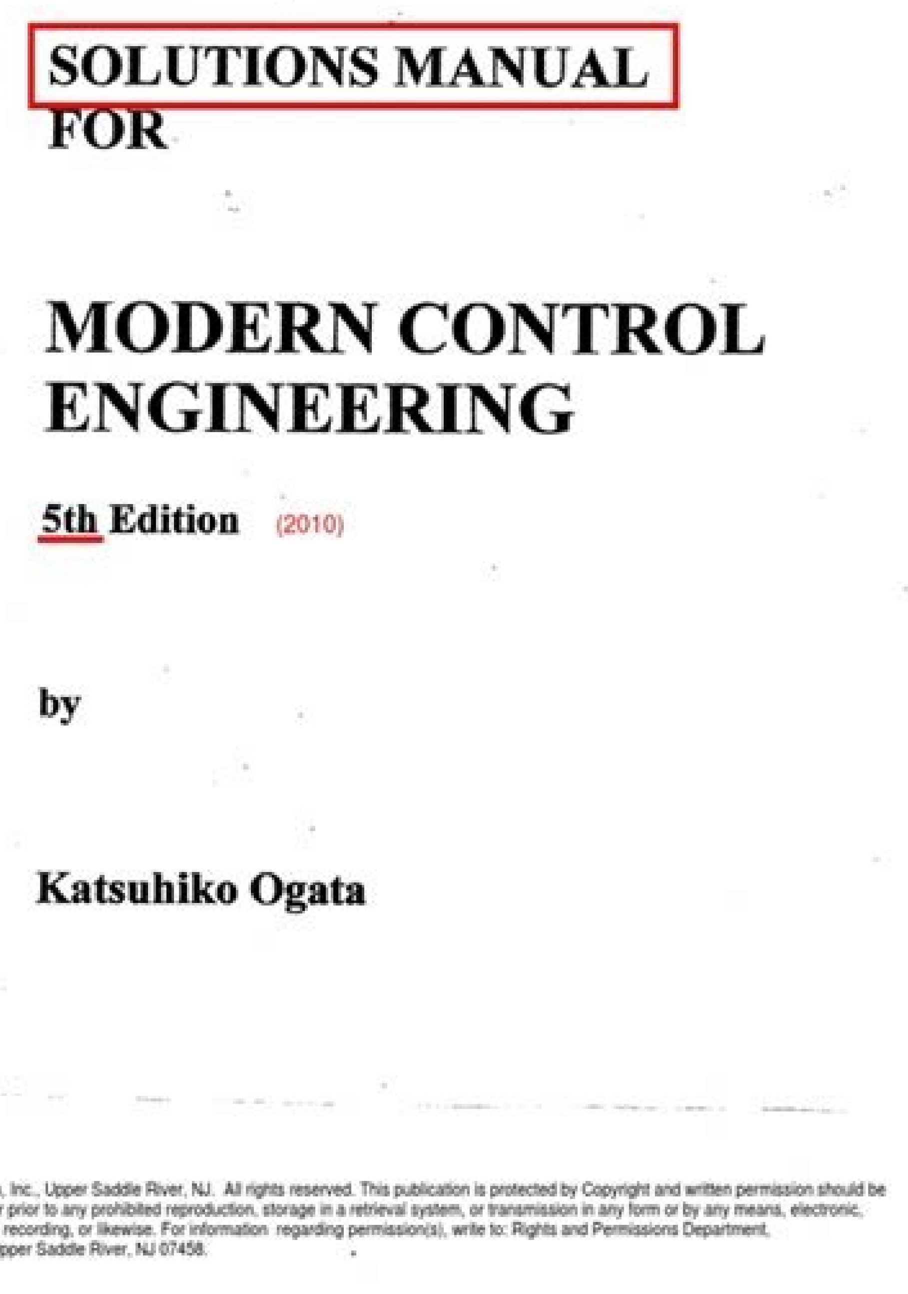

SOLUTIONS MANUAL
FOR

MODERN CONTROL ENGINEERING

5th Edition (2010)

by

Katsuhiko Ogata

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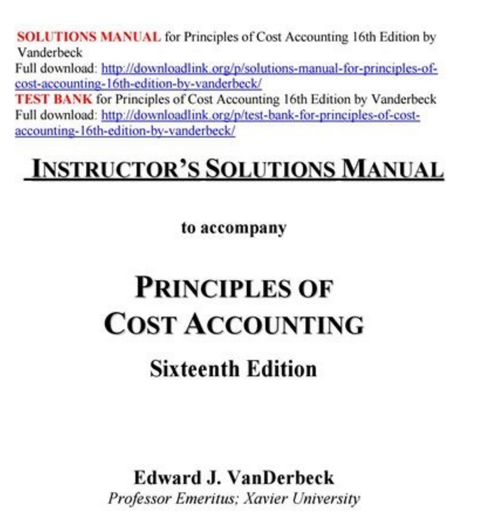

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Preface A course in system dynamics that deals with mathematical modeling and response analyses of dynamic systems is required in most mechanical and other engineering curricula. This book is written as a textbook for such a course. It is written at the junior level and presents a comprehensive treatment of modeling and analyses of dynamic systems and an introduction to control systems. Prerequisites for studying this book are first courses in linear algebra, intro- ductory differential equations, introductory vector-matrix analysis, mechanics, cir- cuit analysis, and thermodynamics.Thermodynamics may be studied simultaneously. Main revisions made in this edition are to shift the state space approach to modeling dynamic systems to Chapter 5, right next to the transfer function approach to modeling dynamic systems, and to add numerous examples for modeling and response analyses of dynamic systems.All plottings of response curves are done with MATLAB. Detailed MATLAB programs are provided for MATLAB works pre- sented in this book. This text is organized into 11 chapters and four appendixes. Chapter 1 presents an introduction to system dynamics. Chapter 2 deals with Laplace transforms of commonly encountered time functions and some theorems on Laplace transform that are useful in analyzing dynamic systems. Chapter 3 discusses details of mechan- ical elements and simple mechanical systems.This chapter includes introductory dis- cussions of work, energy, and power. Chapter 4 discusses the transfer function approach to modeling dynamic sys- tems. 'lransient responses of various mechanical systems are studied and MATLAB is used to obtain response curves. Chapter 5 presents state space modeling of dynam- ic systems. Numerous examples are considered. Responses of systems in the state space form are discussed in detail and response curves are obtained with MATLAB. Chapter 6 treats electrical systems and electromechanical systems. Here we included mechanical-electrical analogies and operational amplifier systems. Chapter 7 vii 8. viii Preface deals with mathematical modeling of fluid systems (such as liquid-level systems, pneumatic systems, and hydraulic systems) and thermal systems. A linearization technique for nonlinear systems is presented in this chapter. Chapter 8 deals with the time-domain analysis of dynamic systems. Transient- response analysis of first-order systems, second-order systems, and higher order sys- tems is discussed in detail. This chapter includes analytical solutions of state-space equations. Chapter 9 treats the frequency-domain analysis of dynamic systems. We first present the sinusoidal transfer function, followed by vibration analysis of mechanical systems and discussions on dynamic vibration absorbers. Then we dis- cuss modes of vibration in two or more degrees-of-freedom systems. Chapter 10 presents the analysis and design of control systems in the time domain. After giving introductory materials on control systems, this chapter discusses transient-response analysis of control systems, followed by stability analysis, root-locus analysis, and design of control systems. Fmally, we conclude this chapter by giving tun- ing rules for PID controllers. Chapter 11 treats the analysis and design of control sys- tems in the frequency domain. Bode diagrams, Nyquist plots, and the Nyquist stability criterion are discussed in detail. Several design problems using Bode diagrams are treated in detail. MATLAB is used to obtain Bode diagrams and Nyquist plots. Appendix A summarizes systems of units used in engineering analyses. Appendix B provides useful conversion tables. Appendix C reviews briefly a basic vector-matrix algebra. Appendix D gives introductory materials on MATLAB. If the reader has no prior experience with MATLAB, it is recommended that he/she study Appendix D before attempting to write MATLAB programs. Throughout the book, examples are presented at strategic points so that the reader will have a better understanding of the subject matter discussed. In addition, a number of solved problems (A problems) are provided at the end of each chapter, except Chapter 1. These problems constitute an integral part of the text. It is sug- gested that the reader study all these problems carefully to obtain a deeper under- standing of the topics discussed. Many unsolved problems (B problems) are also provided for use as homework or quiz problems. An instructor using this text for hislher system dynamics course may obtain a complete solutions manual for B prob- lems from the publisher. Most of the materials presented in this book have been class tested in courses in the field of system dynamics and control systems in the Department of Mechani- cal Engineering, University of Minnesota over many years. If this book is used as a text for a quarter-length course (with approximately 30 lecture hours and 18 recitation hours), Chapters 1 through 7 may be covered. After studying these chapters, the student should be able to derive mathematical models for many dynamic systems with reasonable simplicity in the forms of transfer func- tion or state-space equation. Also, he/she will be able to obtain computer solutions of system responses with MATLAB. If the book is used as a text for a semester- length course (with approximately 40 lecture hours and 26 recitation hours), then the first nine chapters may be covered or, alternatively, the first seven chapters plus Chapters 10 and 11 may be covered. If the course devotes 50 to 60 hours to lectures, then the entire book may be covered in a semester. 9. Preface ix Fmally, I wish to acknowledge deep appreciation to the following professors who reviewed the third edition of this book prior to the preparation of this new edi- tion: R. Gordon Kirk (Vrrginia Institute of Technology), Perry Y. Li (University of Minnesota), Sherif Noah (Texas A & M University), Mark L. Psiaki (Cornell Uni- versity), and William Singhose (Georgia Institute of Technology). Their candid, insightful, and constructive comments are reflected in this new edition. KATSUHIKO OGATA 10. Introduction to System Dynamics 1-1 INTRODUCTION System dynamics deals with the mathematical modeling of dynamic systems and response analyses of such systems with a view toward understanding the dynamic nature of each system and improving the system's performance. Response analyses are frequently made through computer simulations of dynamic systems. Because many physical systems involve various types of components, a wide variety of different types of dynamic systems will be examined in this book. The analysis and design methods presented can be applied to mechanical, electrical, pneumatic, and hydraulic systems, as well as nonengineering systems, such as eco- nomic systems and biological systems. It is important that the mechanical engineer- ing student be able to determine dynamic responses of such systems. We shall begin this chapter by defining several terms that must be understood in discussing system dynamics. Systems. A system is a combination of components acting together to per- form a specific objective.A component is a single functioning unit of a system. By no means limited to the realm of the physical phenomena, the concept of a system can be extended to abstract dynamic phenomena, such as those encountered in eco- nomics, transportation, population growth, and biology. 1 11. 2 Introduction to System Dynamics Chap. 1 A system is called dynamic if its present output depends on past input; if its current output depends only on current input, the system is known as static.The out- put of a static system remains constant if the input does not change. The output changes only when the input changes. In a dynamic system, the output changes with time if the system is not in a state of equilibrium. In this book, we are concerned mostly with dynamic systems. Mathematical models. Any attempt to design a system must begin with a prediction of its performance before the system itself can be designed in detail or ac- tually built. Such prediction is based on a mathematical description of the system's dynamic characteristics. This mathematical description is called a mathematical model. For many physical systems, useful mathematical models are described in terms of differential equations. Linear and nonlinear differential equations. Linear differential equations may be classified as linear, time-invariant differential equations and linear, time- varying differential equations. A linear, time-invariant differential equation is an equation in which a depen- dent variable and its derivatives appear as linear combinations. An example of such an equation is d2x dx - + 5- + lOx =0 dt2 dt Since the coefficients of all terms are constant, a linear, time-invariant differential equation is also called a linear, constant-coefficient differential equation. In the case of a linear, time-varying differential equation, the dependent vari- able and its derivatives appear as linear combinations, but a coefficient or coeffi- cients of terms may involve the independent variable. An example of this type of differential equation is d2 x - + (1 - cos 2t)x = 0 dt2 It is important to remember that, in order to be linear, the equation must con- tain no powers or other functions or products of the dependent variables or its derivatives. A differential equation is called nonlinear if it is not linear. Two examples of nonlinear differential equations are and 12. Sec. 1-2 Mathematical Modeling of Dynamic Systems 3 Linear systems and nonlinear systems. For linear systems, the equations that constitute the model are linear. In this book, we shall deal mostly with linear sys- tems that can be represented by linear, time-invariant ordinary differential equations. The most important property oflinear systems is that the principle ofsuperpo- sition is applicable.This principle states that the response produced by simultaneous applications of two different forcing functions or inputs is the sum of two individual responses. Consequently, for linear systems, the response to several inputs can be calculated by dealing with one input at a time and then adding the results. As a result of superposition, complicated solutions to linear differential equations can be derived as a sum of simple solutions. In an experimental investigation of a dynamic system, if cause and effect are proportional, thereby implying that the principle of superposition holds, the system can be considered linear. Although physical relationships are often represented by linear equations, in many instances the actual relationships may not be quite linear. In fact, a careful study of physical systems reveals that so-called linear systems are actually linear only within limited operating ranges. For instance, many hydraulic systems and pneumatic systems involve nonlinear relationships among their variables, but they are frequently represented by linear equations within limited operating ranges. For nonlinear systems, the most important characteristic is that the principle of superposition is not applicable. In general, procedures for finding the solutions of problems involving such systems are extremely complicated. Because of the mathe- matical difficulty involved, it is frequently necessary to linearize a nonlinear system near the operating condition. Once a nonlinear system is approximated by a linear mathematical model, a number of linear techniques may be used for analysis and design purposes. Continuous-time systems and discrete-time systems. Continuous-time systems are systems in which the signals involved are continuous in time. These sys- tems may be described by differential equations. Discrete-time systems are systems in which one or more variables can change only at discrete instants of time. (These instants may specify the times at which some physical measurement is performed or the times at which the memory of a digital computer is read out.) Discrete-time systems that involve digital signals and, possi- bly, continuous-time signals as well may be described by difference equations after the appropriate discretization of the continuous-time signals. The materials presented in this text apply to continuous-time systems; discrete- time systems are not discussed. 1-2 MATHEMATICAL MODELING OF DYNAMIC SYSTEMS Mathematical modeling. Mathematical modeling involves descriptions of important system characteristics by sets of equations. By applying physical laws to a specific system, it may be possible to develop a mathematical model that describes the dynamics of the system. Such a model may include unknown parameters, which 13. 4 Introduction to System Dynamics Chap. 1 must then be evaluated through actual tests. Sometimes, however, the physical laws governing the behavior of a system are not completely defined, and formulating a mathematical model may be impossible. Ifso, an experimental modeling process can be used. In this process, the system is subjected to a set of known inputs, and its out- puts are measured.Then a mathematical model is derived from the input-output re- lationships obtained. Simplicity of mathematical model versus accuracy of results of analysis. In attempting to build a mathematical model, a compromise must be made between the simplicity of the model and the accuracy of the results of the analysis. It is im- portant to note that the results obtained from the analysis are valid only to the ex- tent that the model approximates a given physical system. In determining a reasonably simplified model, we must decide which physical variables and relationships are negligible and which are crucial to the accuracy of the model. To obtain a model in the form of linear differential equations, any dis- tributed parameters and nonlinearities that may be present in the physical system must be ignored. If the effects that these ignored properties have on the response are small, then the results of the analysis of a mathematical model and the results of the experimental study of the physical system will be in good agreement. Whether any particular features are important may be obvious in some cases, but may, in other instances, require physical insight and intuition. Experience is an important factor in this connection. Usually, in solving a new problem, it is desirable first to build a simplified model to obtain a general idea about the solution.Afterward, a more detailed math- ematical model can be built and used for a more complete analysis. Remarks on mathematical models. The engineer must always keep in mind that the model he or she is analyzing is an approximate mathematical descrip- tion of the physical system; it is not the physical system itself In reality, no mathe- matical model can represent any physical component or system precisely. Approximations and assumptions are always involved. Such approximations and as- sumptions restrict the range of validity of the mathematical model. (The degree of approximation can be determined only by experiments.) So, in making a prediction about a system's performance, any approximations and assumptions involved in the model must be kept in mind. Mathematical modeling procedure. The procedure for obtaining a math- ematical model for a system can be summarized as follows: L Draw a schematic diagram of the system, and define variables. 2. Using physical laws, write equations for each component, combine them according to the system diagram, and obtain a mathematical model. 3. To verify the validity of the model, its predicted performance, obtained by solving the equations of the model, is compared with experimental results. (The question of the validity of any mathematical model can be answered only by experiment.) If the experimental results deviate from the prediction 14. Sec. 1-3 Analysis and Design of Dynamic Systems 5 to a great extent, the model must be modified. A new model is then derived and a new prediction compared with experimental results. The process is re- peated until satisfactory agreement is obtained between the predictions and the experimental results. 1-3 ANALYSIS AND DESIGN OF DYNAMIC SYSTEMS This section briefly explains what is involved in the analysis and design of dynamic systems. Analysis. System analysis means the investigation, under specified condi- tions, of the performance of a system whose mathematical model is known. The first step in analyzing a dynamic system is to derive its mathematical model. Since any system is made up of components, analysis must start by developing a mathematical model for each component and combining all the models in order to build a model of the complete system. Once the latter model is obtained, the analysis may be formulated in such a way that system parameters in the model are varied to produce a number of solutions. The engineer then compares these solutions and interprets and applies the results of his or her analysis to the basic task. H should always be remembered that deriving a reasonable model for the complete system is the most important part of the entire analysis. Once such a model is available, various analytical and computer techniques can be used to ana- lyze it. The manner in which analysis is carried out is independent of the type of physical system involved-mechanical, electrical, hydraulic, and so on. Design. System design refers to the process of finding a system that accom- plishes a given task. In general, the design procedure is not straightforward and will require trial and error. Synthesis. By synthesis, we mean the use of an explicit procedure to find a system that will perform in a specified way. Here the desired system characteristics are postulated at the outset, and then various mathematical techniques are used to synthesize a system having those characteristics. Generally, such a procedure is com- pletely mathematical from the start to the end of the design process. Basic approach to system design. The basic approach to the design of any dynamic system necessarily involves trial-and-error procedures. Theoretically, a synthesis of linear systems is possible, and the engineer can systematically deter- mine the components necessary to realize the system's objective. In practice, howev- er, the system may be subject to many constraints or may be nonlinear; in such cases, no synthesis methods are currently applicable. Moreover, the features of the com- ponents may not be precisely known. Thus, trial-and-error techniques are almost al- ways needed. Design procedures. Frequently, the design of a system proceeds as follows: The engineer begins the design procedure knowing the specifications to be met and 15. 6 Introduction to System Dynamics Chap. 1 the dynamics of the components, the latter of which involve design parameters.The specification may be given in terms of both precise numerical values and vague qualitative descriptions. (Engineering specifications normally include statements on such factors as cost, reliability, space, weight, and ease of maintenance.) It is impor- tant to note that the specifications may be changed as the design progresses, for de- tailed analysis may reveal that certain requirements are impossible to meet. Next, the engineer will apply any applicable synthesis techniques, as well as other meth- ods, to build a mathematical model of the system. Once the design problem is formulated in terms of a model, the engineer car- ries out a mathematical design that yields a solution to the mathematical version of the design problem. With the mathematical design completed, the engineer simu- lates the model on a computer to test the effects of various inputs and disturbances on the behavior of the resulting system. If the initial system configuration is not sat- isfactory, the system must be redesigned and the corresponding analysis completed. This process of design and analysis is repeated until a satisfactory system is found. Then a prototype physical system can be constructed. Note that the process of constructing a prototype is the reverse of mathemati- cal modeling. The prototype is a physical system that represents the mathematical model with reasonable accuracy. Once the prototype has been built, the engineer tests it to see whether it is satisfactory. If it is, the design of the prototype is com- plete. If not, the prototype must be modified and retested. The process continues until a satisfactory prototype is obtained. 1-4 SUMMARY From the point of view of analysis, a successful engineer must be able to obtain a mathematical model of a given system and predict its performance. (The validity of a prediction depends to a great extent on the validity of the mathematical model used in making the prediction.) From the design standpoint, the engineer must be able to carry out a thorough performance analysis of the system before a prototype is constructed. The objective of this book is to enable the reader (1) to build mathematical models that closely represent behaviors of physical systems and (2) to develop sys- tem responses to various inputs so that he or she can effectively analyze and design dynamic systems. Outline of the text. Chapter 1 has presented an introduction to system dy- namics. Chapter 2 treats Laplace transforms. We begin with Laplace transformation of simple time functions and then discuss inverse Laplace transformation. Several useful theorems are derived. Chapter 3 deals with basic accounts of mechanical sys- tems. Chapter 4 presents the transfer-function approach to modeling dynamic sys- tems. The chapter discusses various types of mechanical systems. Chapter 5 examines the state-space approach to modeling dynamic systems. Various types of mechanical systems are considered. Chapter 6 treats electrical systems and electromechanical systems, including operational-amplifier systems. Chapter 7 deals with fluid systems, 16. Sec. 1-4 Summary 7 such as liquid-level systems, pneumatic systems, and hydraulic systems, as well as thermal systems. A linearization technique for nonlinear systems is explored. Chapter 8 presents time-domain analyses of dynamic systems-specifically, transient-response analyses of dynamic systems. The chapter also presents the ana- lytical solution of the state equation. Chapter 9 treats frequency-domain analyses of dynamic systems. Among the topics discussed are vibrations of rotating mechanical systems and vibration isolation problems. Also discussed are vibrations in multi- degrees-of-freedom systems and modes of vibrations. Chapter 10 presents the basic theory of control systems, including transient- response analysis, stability analysis, and root-locus analysis and design. Also dis- cussed are tuning rules for PID controllers. Chapter 11 deals with the analysis and design of control systems in the frequency domain. The chapter begins with Bode diagrams and then presents the Nyquist stability criterion, followed by detailed design procedures for lead, lag, and lag-lead compensators. Appendix A treats systems of units, Appendix B summarizes conversion tables, and Appendix C gives a brief summary of vector-matrix algebra. Appendix D presents introductory materials for MATLAB. Throughout the book, MATLAB is used for the solution of most computa- tional problems. Readers who have no previous knowledge of MATLAB may read Appendix D before solving any MATLAB problems presented in this text. 17. The Laplace Transform 2-1 INTRODUCTION The Laplace transform is one of the most important mathematical tools available for modeling and analyzing linear systems. Since the Laplace transform method must be studied in any system dynamics course, we present the subject at the begin- ning of this text so that the student can use the method throughout his or her study of system dynamics. The remaining sections of this chapter are outlined as follows: Section 2-2 reviews complex numbers, complex variables, and complex functions. Section 2-3 defines the Laplace transformation and gives Laplace transforms of several com- mon functions of time. Also examined are some of the most important Laplace transform theorems that apply to linear systems analysis. Section 2-4 deals with the inverse Laplace transformation. Finally, Section 2-5 presents the Laplace transform approach to the solution of the linear, time-invariant differential equation. 2-2 COMPLEX NUMBERS, COMPLEX VARIABLES, AND COMPLEX FUNCTIONS This section reviews complex numbers, complex algebra, complex variables, and complex functions. Since most of the material covered is generally included in the basic mathematics courses required of engineering students, the section can be omitted entirely or used simply for personal reference. S 18. Sec. 2-2 Complex Numbers, Complex Variables, and Complex Functions 1m o Re Figure 2-1 Complex plane representa- tion of a complex number z. 9 Complex numbers. Using the notation j = v=I, we can express all num- bers in engineering calculations as z = x + jy where z is called a complex number and x and jy are its real and imaginary parts, respectively. Note that both x and y are real and that j is the only imaginary quanti- ty in the expression. The complex plane representation of z is shown in Figure 2-1. (Note also that the real axis and the imaginary axis define the complex plane and that the combination of a real number and an imaginary number defines a point in that plane.) A complex number z can be considered a point in the complex plane or a directed line segment to the point; both interpretations are useful. The magnitude, or absolute value, of z is defined as the length of the directed line segment shown in Figure 2-1. The angle of z is the angle that the directed line segment makes with the positive real axis. A counterclockwise rotation is defined as the positive direction for the measurement of angles. Mathematically, magnitude of z = Izl = Vx2 + j, angle of z = 9 = tan-1l:'. x A complex number can be written in rectangular form or in polar form as follows: z = x + jy z = Izl(cos 9 + j sin 9) z= Izl~ z = Izl eifJ }rectangular fonns }polar forms In converting complex numbers to polar form from rectangular, we use Izl = Vx2 + y2, 8 = tan-II x To convert complex numbers to rectangular form from polar, we employ x = IzIcos 8, y = Izl sin 8 Complex conjugate. The complex conjugate of z = x + j y is defined as Z = x - jy 19. 10 The Laplace Transform Chap. 2 Figure 2-2 Complex number zand its complex conjugate Z, 1m o Re The complex conjugate ofzthus has the same real part as Z and an imaginary part that is the negative of the imaginary part of z. Figure 2-2 shows both zand Z. Note that z = x + jy = Izl il = Izi (cos 8 + j sin 8) z = x - jy = Izi /-8 = Izi (cas 8 - jsin8) Euler's theorem. The power series expansions of cos 8 and sin 8 are, respectively, and Thus, Since fi2 £t 86 cos 8 = 1 - - + - - - + 2! 4! 6! . 83 85 87 sm8 = 8 - - + - - - + 3! 5! 7! ('8)2 ('8)3 ('8)4 8 + ' , 8 = 1 + ('8) + -'- + -'- + -'- +cos ,sm , , 3' 4'2. . . it follows that cos 8 + j sin 8 = ejB This is known as Euler's theorem. Using Euler's theorem, we can express the sine and cosine in complex form. Noting that e-jB is the complex conjugate of ei6 and that eiB = cos 8 + j sin 8 e-jB = cos 8 - j sin 8 I 20. Sec. 2-2 Complex Numbers, Complex Variables, and Complex Functions 11 we find that ej8 + e-j8 cos 8 = 2 . ej8 - e-j8 sm8 = 2j Complex algebra. If the complex numbers are written in a suitable form, op- erations like addition, subtraction, multiplication, and division can be performed easily. Equality ofcomplex numbers. 1vo complex numbers z and ware said to be equal if and only if their real parts are equal and their imaginary parts are equal. So if two complex numbers are written z = x + jy, w = u + jv then z = w if and only if x = u and y = v. Addition. 1vo complex numbers in rectangular form can be added by adding the real parts and the imaginary parts separately: z + w = (x + jy) + (u + jv) = (x + u) + j(y + v) Subtraction. Subtracting one complex number from another can be consid- ered as adding the negative of the former: z - w = (x + jy) - (u + jv) = (x - u) + j(y - v) Note that addition and subtraction can be done easily on the rectangular plane. Multiplication. If a complex number is multiplied by a real number, the re- sult is a complex number whose real and imaginary parts are multiplied by that real number: az = a(x + jy) = ax + jay (a = real number) Iftwo complex numbers appear in rectangular form and we want the product in rec- tangular form, multiplication is accomplished by using the fact that P = -1. Thus, if two complex numbers are written z = x + jy, w = u + jv then zw = (x + j y)(u + jv) = xu + j yu + jxv + lyv = (xu - yv) + j(xv + yu) In polar form, multiplication of two complex numbers can be done easily. The mag- nitude of the product is the product of the two magnitudes, and the angle of the product is the sum of the two angles. So if two complex numbers are written z = Izl~, w = Iwl~ then zw = Izllwl/8 + cP 21. 12 The Laplace Transform Chap. 2 Multiplication byJ. It is important to note that multiplication by j is equiva- lent to counterclockwise rotation by 90°. For example, if z = x + jy then jz = j(x + jy) = jx + py = -y + jx or, noting that j = 1/90°, if z = Izl il then jz = 1/90° Izl il = Izl/8 + 90° Figure 2-3 illustrates the multiplication of a complex number z by j. Division. H a complex number z = IzIil is divided by another complex number w = IwIil., then z Izi il Izi w = Iwl L.!2. = M 18 - ~ That is, the result consists of the quotient of the magnitudes and the difference of the angles. Division in rectangular form is inconvenient, but can be done by mUltiplying the denominator and numerator by the complex conjugate of the denominator.This procedure converts the denominator to a real number and thus simplifies division. For instance, z x + jy (x + jy)(u - jv) (xu + yv) + j(yu - xv) - = - - = = ~--~--.;...~--..;... w u + jv (u + jv) (u - jv) u2 + v2 xu + yv + .yu - xv = u 2 + v 2 J u2 + v 2 o Figure 2-3 Multiplication of a complex number z by j. Re 1m o Re Figure 2-4 Division of a complex number z by j. 22. Sec. 2-2 Complex Numbers, Complex Variables, and Complex Functions 13 Division by j. Note that division by j is equivalent to clockwise rotation by 90°. For example, if z = x + jy, then z x + jy (x + jy)j jx - y . -=--= =--=y-]X j j jj -1 or z Izl L.P. j = 1 /90° = Izl /8 - 90° Figure 2-4 illustrates the division of a complex number z by j. Powers and roots. Multiplying z by itself n times, we obtain zn = (Izl L.P.)n = Izln / n8 Extracting the nth root of a complex number is equivalent to raising the number to the 1/nth power: For instance, and (8.66 - j5)3 = (10 /-30°)3 = 1000 /-90° = 0 - j 1000 = -j 1000 (2.12 - j2.12)112 = (9 / -45°)112 = 3 / -22.5° Comments. It is important to note that Izwl = Izllwl Iz + wi #: Izi + Iwl Complex variable. A complex number has a real part and an imaginary part, both of which are constant. If the real part or the imaginary part (or both) are variables, the complex number is called a complex variable. In the Laplace transfor- mation, we use the notation s to denote a complex variable; that is, s = u + jw where u is the real part and jw is the imaginary part. (Note that both u and ware real.) Complex function. A complex function F(s), a function of s, has a real part and an imaginary part, or F(s) = Fx + jFy where Fx and Fy are real quantities. The magnitude of F(s) is VFi + F~, and the angle 8 of F(s) is tan-1 (FylFx)' The angle is measured counterclockwise from the positive real axis.The complex conjugate of F(s) is pes) = Fx - jFy- Complex functions commonly encountered in linear systems analysis are single- valued functions of s and are uniquely determined for a given value of s.1}rpically, 23. 14 The Laplace Transform Chap. 2 such functions have the form K(s + ZI)(S + Z2) ... (s + Zm) F(s) - ---:...-..---.;...~----- - (s + PI)(S + P2) ... (s + Pn) Points at which F(s) equals zero are called zeros. That is, s = -Zh S = -Z2, ... , s = -Zm are zeros of F(s). [Note that F(s) may have additional zeros at infinity; see the illustrative example that follows.] Points at which F(s) equals infinity are called poles. That is, s = -PI, S = - P2, ... , s = - Pn are poles of F(s). If the denominator of F(s) involves k-multiple factors (s + Pl, then s = -pis called a multiplepole of order k or repeated pole of order k.1f k = 1, the pole is called a simple pole. As an illustrative example, consider the complex function JC(s + 2)(s + 10) G(s) - --------~ - s(s + l)(s + 5)(s + 15)2 G(s) has zeros at s = -2 and s = -10, simple poles at s = 0, s = -1, and s = -5, and a double pole (multiple pole of order 2) at s = -15. Note that G(s) becomes zero at s = 00. Since, for large values ofs, K G(s) * 3s it follows that G(s) possesses a triple zero (multiple zero of order 3) at s = 00. If points at infinity are included, G(s) has the same number of poles as zeros. To sum- marize, G(s) has five zeros (s = -2, s = -10, s = 00, s = 00, s = 00) and five poles (s = 0,

s = -1, s = -5, s = -15, s = -15). 2-3 LAPLACE TRANSFORMATION The Laplace transform method is an operational method that can be used advanta- geously in solving linear, time-invariant differential equations. Its main advantage is that differentiation of the time function corresponds to multiplication of the trans- form by a complex variable s, and thus the differential equations in time become algebraic equations in s.The solution of the differential equation can then be found by using a Laplace transform table or the partial-fraction expansion technique. Another advantage of the Laplace transform method is that, in solving the differen- tial equation, the initial conditions are automatically taken care of, and both the par- ticular solution and the complementary solution can be obtained simultaneously. Laplace transformation. Let us define /(t) = a time function such that /(t) =°for t < 0 s = a complex variable 9!, = an operational symbol indicating that the quantity upon which it operates is to be transformed by the Laplace integral100 e-st dt F(s) = Laplace transform off(t) 24. Sec. 2-3 Laplace Transformation 15 Then the Laplace transform off(t) is given by ~[f(t)] = F(s) = l"'e-n dt[f(t)] = l""f(t)e-n dt The reverse process of finding the time function f(t) from the Laplace transform F(s) is called inverse Laplace trans/ormation.The notation for inverse Laplace trans- formation is ;r1. Thus, ;rl[F(s)] = /(t) Existence of Laplace transform. The Laplace transform of a function f(t) exists if the Laplace integral converges. The integral will converge iff(t) is piecewise continuous in every finite interval in the range t > 0 and if I(t) is of exponential order as t approaches infinity. A function f(t) is said to be of exponential order if a real, positive constant u exists such that the function e-atl/(t)I approaches zero as t approaches infinity. If the limit of the function e-utl/(t)I approaches zero for u greater than uc and the limit approaches infinity for u less than uC' the value uc is called the abscissa 0/convergence. It can be seen that, for such functions as t, sin wt, and t sin wt, the abscissa of convergence is equal to zero. For functions like e-ct , te-ct , and e-ct sin wt, the abscis- sa of convergence is equal to -c. In the case of functions that increase faster than the exponential function, it is impossible to find suitable values of the abscissa of convergence. Consequently, such functions as il and ter do not possess Laplace transforms. Nevertheless, it should be noted that, although er for 0 s t S 00 does not possess a Laplace transform, the time function defined by /(t) = er =0 for 0 :s; t :s; T < 00 for t < 0, T < t does, since / (t) = er for only a limited time interval 0 S t !5 T and not for oS t S 00. Such a signal can be physically generated. Note that the signals that can be physically generated always have corresponding Laplace transforms. If functions 11(t) and h(t) are both Laplace transformable, then the Laplace transform of11(t) + h(t) is given by ;e[fl(t) + h(t)] = ;e[f1(t)] + ;e[f2(t)] Exponential function. Consider the exponential function / (t) = 0 for t < 0 =Ae-at for t ~ 0 where A and a are constants. The Laplace transform of this exponential function can be obtained as follows: 1 00 100 A;e[Ae-at] = Ae-ate-st dt = A e-(a+s)t dt = - - o 0 s + a 25. 16 The Laplace Transform Chap. 2 In performing this integration, we assume that the real part of s is greater than -a (the abscissa of convergence), so that the integral converges. The Laplace trans- form F(s} of any Laplace transformable functionf(t) obtained in this way is valid throughout the entire s plane, except at the poles of F(s). (Although we do not pre- sent a proof of this statement, it can be proved by use of the theory of complex variables.) Step function. Consider the step function f (t) = 0 for t < 0 = A fort> 0 where A is a constant. Note that this is a special case of the exponential function Ae-at, where a = O. The step function is undefined at t = O. Its Laplace transform is given by 1 00 A ;£[A] = Ae-st dt = - o s The step function whose height is unity is called a unit-step function. The unit- step function that occurs at t = to is frequently written l(t - to), a notation that will be used in this book.The preceding step function whose height is A can thus be writ- tenA1(t}. The Laplace transform of the unit-step function that is defined by is lis, or l(t) = 0 =1 for t < 0 for t > 0 ~[l(t)] = !s Physically, a step function occurring at t =to corresponds to a constant signal suddenly applied to the system at time t equals to. Ramp function. Consider the ramp function f(t) = 0 = At for t < 0 for t ~ 0 where A is a constant.The Laplace transform of this ramp function is !'eIAt] = A ["'te-Sf dt To evaluate the integral, we use the formula for integration by parts: [budv =Uvl: - [bVdU 26. Sec. 2-3 Laplace Transformation In this case, U =t and dv =e-st dt. [Note that v = e-SII(-s).] Hence, 100 (-sl I00 100 -Sf) ;e[At] = A te-sr dt = A t ~ - ~dt o -s 0 o-s = A fooe-sr dt = A s 10 s2 Sinusoidal function. The Laplace transform of the sinusoidal function J(t) = 0 for t < 0 = A sin wt for t ~ 0 where A and ware constants, is obtained as follows: Noting that ejw1 = cos wt + j sin wt and e-jwt = cos wt - j sin wt we can write Hence, AlOO • •~[A sin wt] = --: (eJwt - e-Jwt)e-st dt 2J 0 Al A1 Aw = - - - - - - - = 2j s - jw 2j s + jw s2 + w2 Similarly, the Laplace transform of A cos wt can be derived as follows: As ;erA cos wt] = 2 2 S + w 17 Comments. The Laplace transform of any Laplace transformable function f(t) can be found by multiplying f(t) by e-st and then integrating the product from t = 0 to t = 00. Once we know the method of obtaining the Laplace transform, how- ever, it is not necessary to derive the Laplace transform of I(t) each time. Laplace transform tables can conveniently be used to find the transform of a given function f(t). Table 2-1 shows Laplace transforms of time functions that will frequently appear in linear systems analysis. In Table 2-2, the properties ofLaplace transforms are given. Translated function. Let us obtain the Laplace transform of the translated function f(t - a)l(t - a), where a ~ O. This function is zero for t < a. The func- tionsf(t)l(t) and f(t - a)1(t - a) are shown in Figure 2-5. By definition, the Laplace transform of J(t - a)l(t - a) is ~1f(1 - a)t(t - a)] = [,oJ(1 - a)t(1 - aV" dl 27. 18 The Laplace Transform Chap. 2 TABLE 2-1 Laplace Transform Pairs f(t) F(s) 1 Unit impulse cS{t) 1 2 Unit step 1(t) 1 - s 1 3 t s2 4 tn- 1 (n=1,2,3, ...) 1 - (n - 1)! sn 5 tn (n=1,2,3, ...) n! -sn+l 6 e-at 1 --s+a 1 7 te-at (s + a)2 1 n-l -at (n = 1, 2, 3, ...) 1 8 (n - 1)! t e (s + a)n 9 tne-at (n=1,2,3, ...) n! (s + a)n+l w 10 sin wt $2 + w2 S 11 coswt s2 + w2 w 12 sinh wt S2 - (J)2 $ 13 cosh wt ; - w2 14 !(1 - e-at) 1 a s(s + a) 15 _1_(e-at _ e-bt) 1 b-a (s + a)(s + b) _1_(be-bt - ae-at) s 16 (s + a)(s + b)b-a ~[1 + _1_(be-at - ae-bt)] 1 17 s(s + a)(s + b)ab a - b 28. TABLE 2-1 (continued) r I 1-- f(t) I lpes) - 18 1 2 (1 e 01 a ate 01) 1 s(s + a)2 - 19 1 '--- 2 (at 1 + e-al) 1 a s2(s + a) -w 20 e 01 sin wt - (s + a)2 + ;;;. ~ 21 l.- e 01 cos wt s+a - (s + a)2 +-;;; - 22 Wn_e(wnt· ~~ smwn 1-,2 t w2 n 2 s + 2,wns + w2 n - 1- =e (Wnl' (~ 23 ~ smwn 1-,2 / -cJ» cJ> = tan-1YI -,2 _s 2 S + 2,wns + w2 , - n - 1 1 __e(wnt· (~ 24 ~ smwn 1-,2 / +cJ» cJ> = tan-l !l -,2 w2 n S(S2 + 2,wns + w1) ' - - w2 25 1 cos wt -S(S2 + w2) ' - - w3 26 wt - sin wt - s2(s2 + (2) - 2w3 27 sin wt - wt cos wt - (s2 + w2)2 - 28 1 2wI sin wI s - (s2 + w2)2 - s2 - w2 29 t cos wt - (s2 + (2)2 - - I _ 1 30 ~ _:I(cos WIt cos i»2t) (WI ¢~) S (s2 + wI)(s2 + ~) - 31 1 2w (sin wt + wt cos wt) - S2 (s2 + (2)2 L.. 19 29. TABLE 2-2 Properties of Laplace Transforms 1 ~[Af(t)] :::: AF(s) 2 ~[fl(t) ± 12(t)] :::: P1(s) ± F2(S) 3 ~±[:tf(t)] = sF(s) - f(O±) 4 !:e±[:~f(t)] = s2p(s) - sf(O±) - i(O±) [dn] n (k-l) ~± dtnf(t) = snp(s) - ~sn-k f(O±) 5 (k-l) dk- 1 where f(t) :::: dtk-1f(t) 6 [J] F(s) If[(I) dll/eD< ~± f(t)dt = - + s s [f!] F(s) If[(I) dll/eD< Iff[(I) dl dll/eD< 7 ~± f(t) dt dt = - 2 + 2 + S S s 8 ~±[/···1f(t)(dt)n] = F(:) + ±n_ 1 k+l [/···1f(t)(dt)k] s k=l S I=O± 9 f£[!o'[(I) dl] = F~S) 10 ["[(I) dl = lim F(s) if1.""[(I) dl exists o s-O 11 ~[e-a'f(t)] = F(s + a) 12 !:e[f(t - a)l(t - a)] = e-asF(s) a~O 13 dP(s) !:e[tf(t)] = - - - ds 14 d2 ~[t2f(t)] = -2 F(s) ds dn 15 ~[tnf(t)] = (-l)n-F(s) n = 1,2,3, ... dsn 16 f£[7[(I)] = 1""F(S) ds if lim.!.f(t) exists I-ot 17 ~[f(~)] = aF(as) 20 30. Sec. 2-3 Laplace Transformation f(t) l(t) f(1 - DC) l(t - DC) o o DC Figure 2-S Functionf(t)l(t) and translated function f(t - a)l(1 - a). By changing the independent variable from t to 7, where 7 = t - a, we obtain foof(t - £1')l(t - £1')e-st dt = l°Of (7)1(T)e-S(T+a) dT 10 -a 21 Noting that f(T)1(7) = 0 for T < 0, we can change the lower limit of integration from -a to O.Thus, l°O f (T)l(T)e-S(T+a> dT = f OO f (7)1(T)e-S(T+a) dT -a 10 =["'j(T)e-STe-as dT =e-as100 j(T)e-ST dT =e-asp(s) where P(s) = ~(f(t)] = 1°Oj (tv" dl Hence, ;Eff(t - a)l(t - a)] = e-aSF(s) a~O This last equation states that the translation of the time functionf(t)l(t) by a (where a ~ 0) corresponds to the multiplication of the transform F(s) bye-as. Pulse function. Consider the pulse function shown in Figure 2-6, namely, A f(t) =- to =0 where A and to are constants. for 0 < t < to for t < 0, to < t The pulse function here may be considered a step function of height Alto that begins at t = 0 and that is superimposed by a negative step function of height Alto 31. 22 The Laplace Transform Chap. 2 1(1) A to o 4_ 00 to Figure 2-6 Pulse function. Figure 2-7 Impulse function. beginning at t = to; that is, A A f(t) = -1(t) - -1(t - to) to to Then the Laplace transform off(t) is obtained as ~[J(t)l = ~[~1(t)] - ~[~1(t - to)] A A -SI = - - -e 0 tos tos A _ = -(1 - e stO) tos (2-1) Impulse function. The impulse function is a special limiting case of the pulse function. Consider the impulse function f (t) = lim A for 0 < t < to-O to = 0 for t < 0, to < t Figure 2-7 depicts the impulse function defined here. It is a limiting case ofthe pulse function shown in Figure 2-6 as to approaches zero. Since the height of the impulse function is Alto and the duration is to, the area under the impulse is equal to A. As the duration to approaches zero, the height Alto approaches infinity, but the area under the impulse remains equal to A. Note that the magnitude of the impulse is measured by its area. From Equation (2-1), the Laplace transform of this impulse function is shown to be :Eff(t)] = lim [~(1 -e-sto)] 10-0 tos ~[A(l - e-S1o)] . dto As =hm =-=A 10-0 d () s - tos dto 32. Sec. 2-3 Laplace Transformation 23 Thus, the Laplace transform of the impulse function is equal to the area under the impulse. The impulse function whose area is equal to unity is called the unit-impulse function or the Dirac delta function. The unit-impulse function occurring at t = to is usually denoted by eS(t - to), which satisfies the following conditions: eS(t - to) = 0 eS(t - to) = 00 J~6(1 - '0) dl = 1 for t *to for t = to An impulse that has an infinite magnitude and zero duration is mathematical fiction and does not occur in physical systems. If, however, the magnitude of a pulse input to a system is very large and its duration very short compared with the system time constants, then we can approximate the pulse input by an impulse function. For instance, if a force or torque input f(t) is applied to a system for a very short time duration 0 < t < to, where the magnitude of f(t) is sufficiently large so that J;o f(t) dt is not negligible, then this input can be considered an impulse input. (Note that, when we describe the impulse input, the area or magnitude of the impulse is most important, but the exact shape of the impulse is usually immaterial.) The impulse input supplies energy to the system in an infInitesimal time. The concept of the impulse function is highly useful in differentiating discon- tinuous-time functions. The unit-impulse function eS(t - to) can be considered the derivative of the unit-step function l(t - to) at the point of discontinuity t = to, or d eS(t - to) = dt l(t - to) Conversely, if the unit-impulse function eS(t - to) is integrated, the result is the unit- step function l(t - to). With the concept of the impulse function, we can differenti- ate a function containing discontinuities, giving impulses, the magnitudes of which are equal to the magnitude of each corresponding discontinuity. Multiplication of fIt) bye-at. If f(t) is Laplace transformable and its Laplace transform is F(s), then the Laplace transform of e-at f(t) is obtained as ~[e"""f(l)l = [X>e-alf(t)e-st dl = F(s + a) (2-2) We see that the multiplication of f(t) bye-at has the effect of replacing s by (s + a) in the Laplace transform. Conversely, changing s to (s + a) is equivalent to mUltiplyingf(t) bye-at. (Note that a may be real or complex.) The relationship given by Equation (2-2) is useful in finding the Laplace transforms of such functions as e-at sin wt and e-at cos wt. For instance, since ~[sin wt] = 2 W 2 = F(s) s + w and s ~[cos wt] = 2 2 = G(s) s + w 33. 24 The Laplace Transform Chap. 2 it follows from Equation (2-2) that the Laplace transforms of e-at sin wt and e-at cos wt are given, respectively, by and w !e[e-al sin wt] = F(s + a) = - - - - - (s + a)2 + w2 s+a !e[e-al cos wt] = G(s + a) = -----::- (s + a)2 + w2 Comments on the lower limit of the Laplace integral. In some cases,Jtt) possesses an impulse function at t = O. Then the lower limit of the Laplace integral must be clearly specified as to whether it is 0- or 0+, since the Laplace transforms ofJtt) differ for these two lower limits. If such a distinction of the lower limit of the Laplace integral is necessary, we use the notations and .:£_[f(t)] = ["'[(t)e-n dt = .:£+[f(t)] + [0+[(t)e-st dt IfJtt) involves an impulse function at t = 0, then since [[(t)e-st dt .. 0 for such a case. Obviously, ifJtt) does not possess an impulse function at t = 0 (i.e., if the function to be transformed is finite between t = 0- and t = 0+), then !e+[f(t)] = !e_[f(t)] Differentiation theorem. The Laplace transform of the derivative of a function/(t) is given by .:£[:/(t)] = sF(s) - [(0) (2-3) where .f(0) is the initial value of.f(t), evaluated at t = O. Equation (2-3) is called the differentiation theorem. For a given function .f(t), the values of /(0+) and 1(0-) may be the same or different, as illustrated in Figure 2-8. The distinction between 1(0+) and 1(0-) is important when.f(t) has a discontinuity at t = 0, because, in such a case, dJtt)/dt will 34. Sec. 2-3 Laplace Transformation /(/) /(0 +) /(/) Flgure 2-8 Step function and sine function indicating initial values at 1 = 0- and 1 = 0+. 25 involve an impulse function at t = O. If 1(0+) ::F 1(0-), Equation (2-3) must be modified to ~+[:,t(t)] = sF(s) - [(0+) ~-[:,t(t)]= sF(s) - /(0-) To prove the differentiation theorem, we proceed as follows: Integrating the Laplace integral by parts gives roo -st I00 (:JO[d] -sf 10 I(t)e- st dt = I(t) ~s 0- 10 dtl(t) ~s dt Hence, 1(0) 1 [d]F(s) = - + -;£ -/(t) s s dt It follows that ~[:t[(t)]= sF(s) - [(0) Similarly, for the second derivative ofI(t), we obtain the relationship ~[;:[(t)]= h(s) - s/(O) - j(O) where 1(0) is the value of dl(t)ldt evaluated at t = O. To derive this equation, define d dtl(t) = g(t) 35. 26 Then The Laplace Transform Chap. 2 !'£[:I:/(/)] =!'£[:lg(/)] = s!'£[g(t)] - g(O) = S!'£[:,t(/)] - j(O) =l-P(s) - sl(O) - i(O) Similarly, for the nth derivative of f(t), we obtain !'£[:/:t 0, and if s2 + as + b = 0 has a pair of complex-conjugate roots, then expand F(s) into the following partial-fraction expansion form: c ds + e F(s) = -; + -s2-+- a - s -+-b (See Example 2-3 and Problems A-2-15, A-2-16, and A-2-19.) ExampJe2-3 Find the inverse Laplace transform of F(s) = 2s + 12 s2 + 2s + 5 Notice that the denominator polynomial can be factored as s2 + 2s + S = (s + 1 + j2)(s + 1 - j2) The two roots of the denominator are complex conjugates. Hence, we expand F(s) into the sum of a damped sine and a damped cosine function. Noting that; + 2s + S = (s + 1)2 + 22 and referring to the Laplace trans- forms of e-al sin wt and e-al cos Cdt, rewritten as and _ s+a .'i[e at cos Cdt] = 2 2 (s + a) + w we can write the given F(s) as a sum of a damped sine and a damped cosine function: 2s + 12 10 + 2(s + 1) F(s) - - - - - - s2 + 2s + S (s + 1)2 + 22 =S 2 +2 s+1 (s + 1)2 + 22 (s + 1)2 + 22 It follows that f(t) = .'i-I[F(s)] = SX- 1 [(S + 1~2 + 22] + 2!C 1 [(S +S1;21+ 22] = Se-1 sin 2t + 2e-1 cos 2t t ~ 0 Partial- fraction expansion when F(s) involves multiple poles. Instead of discussing the general case, we shall use an example to show how to obtain the partial-fraction expansion of F(s). (See also Problems A-2-17 and A-2-19.) Consider F (s) = s2 + 2s + 3 (s + 1)3 42. Sec. 2-4 Inverse Laplace Transformation 33 The partial-fraction expansion of this F(s) involves three terms: F(B(s) ~ ~ b1 s) = A(s) = (s + 1? + (s + 1)2 + S + 1 where b3, b2, and b1 are determined as follows: Multiplying both sides of this last equation by (s + 1?, we have B(s) (s + 1)3 A(s) = ~ + b,,(s + 1) + b1(s + 1)2 (2-8) Then, letting s = -1, we find that Equation (2-8) gives [(s + 1)3 B (S)] = ~ A(s) s=-1 Also, differentiating both sides of Equation (2-8) with respect to s yields :s [(s + 1)3 !~:~] = ~ + 2bt(s + 1) (2-9) H we let s = -1 in Equation (2-9), then ![(S + 1?B(S)] = ~ ds A(s) s=-1 Differentiating both sides of Equation (2-9) with respect to s, we obtain ~[(S + 1)3 B (S)] = 2b1ds2 A(s) From the preceding analysis, it can be seen that the values of ~, ~, and b1 are found systematically as follows: ~ = [(s + 1)3!~:~L-I = (s2 + 2s + 3)S=-1 =2 ~ = {:s [(s + 1)3 !~:~]L-I = [~(S2 + 2s + 3)]ds s=-1 = (2s + 2)s=-1 =0 b1= ~ {if.[(S + 1)3 B(S)]} 2! ds2 A(s) s=-1 = J,[d 2 2(s2 + 2s + 3)]2. ds s=-1 =1.(2) = 1 2 43. 34 The Laplace Transform Chap. 2 We thus obtain f(t) = ~-l[F(s)] _;e-l[2] + ;e-1[0] + ;e-l[_1_]- (s + 1)3 (s + 1)2 S + 1 = t2e-t + 0 + e-t = (t2 + 1)e-t t ~ 0 2-5 SOLVING LINEAR, TIME-INVARIANT DIFFERENTIAL EQUATIONS In this section, we are concerned with the use of the Laplace transform method in solving linear, time-invariant differential equations. The Laplace transform method yields the complete solution (complementary solution and particular solution) of linear, time-invariant differential equations. Classical methods for finding the complete solution of a differential equation require the evaluation of the integration constants from the initial conditions. In the case of the Laplace transform method, however, this requirement is unnecessary because the initial conditions are automatically included in the Laplace transform of the differential equation. If all initial conditions are zero, then the Laplace transform of the differential equation is obtained simply by replacing dldt with s, d21dt2 with s2, and so on. In solving linear, time-invariant differential equations by the Laplace trans- form method, two steps are followed: 1. By taking the Laplace transform of each term in the given differential equa- tion, convert the differential equation into an algebraic equation in s and ob- tain the expression for the Laplace transform of the dependent variable by rearranging the algebraic equation. 2. The time solution of the differential equation is obtained by rmding the in- verse Laplace transform of the dependent variable. In the discussion that follows, two examples are used to demonstrate the solu- tion of linear, time-invariant differential equations by the Laplace transform method. Example 2-4 Find the solution x(t) of the differential equation :i + 3x + 2x = 0, x(O) = a, x(O) = b where a and b are constants. Writing the Laplace transform ofx(t) as Xes), or ;e[x(t)] = Xes) we obtain !e[x] = sX(s) - x(O) !e[:i] = s2Xes) - sx(O) - x(O) 44. Sec. 2-5 Solving Linear, lime-Invariant Differential Equations The Laplace transform of the given differential equation becomes [s2X(s) - sx(O) - x(O)] + 3[sX(s) - x(O)] + 2X(s) = 0 Substituting the given initial conditions into the preceding equation yields [s2X(s) - as - b] + 3[sX(s) - a] + 2X(s) = 0 or (S2 + 3s + 2)X(s) = as + b + 3a Solving this last equation for X(s), we have X (s) = as + b + 3a = as + b + 3a = _2o_+_b _ _a_+_b s2 + 3s + 2 (s + 1)(s + 2) s + 1 s + 2 The inverse Laplace transform of X(s) produces x(t) = ~-l[X(S)] = ~-1[2a + b]_ ;£-I[a + b] s+1 s+2 = (20 + b)e-I - (a + b)e-2t t ~ 0 35 which is the solution of the given differential equation. Notice that the initial condi- tions a and b appear in the solution. Thus,x(t) has no undetermined constants. Example 2-5 Find the solution x(t) of the differential equation x + 2X + 5x = 3, x(O) = 0, x(O) = 0 Noting that ;£[3] = 3/s, x(O) = 0, and X(O) = 0, we see that the Laplace trans- form of the differential equation becomes 3 S2X(s) + 2sX(s) + 5X(s) = - s Solving this equation for X(s), we obtain 3 X(s) = s(S2 + 2s + 5) 31 3 s+2 =-- - 5 s 5 s2 + 2s + 5 31 3 2 5s Hence, the inverse Laplace transform becomes x(t) = ;£-l[X(s)) 3 s + 1 5(s+I)2+22 -~;£-1[!]_ ~~-1[2]_ ~;£-1[S + 1] - 5 s 10 (s + 1)2 + 22 5 (s + 1)2 + 22 3 3 -I • 2 3 -I 2= - - - e SID t - - e cos t 5 10 5 t~O which is the solution of the given differential equation. 45. 36 The Laplace Transform Chap. 2 EXAMPLE PROBLEMS AND SOLUTIONS Problem A-2-1 Obtain the real and imaginary parts of 2 + j1 3 + j4 Also, obtain the magnitude and angle of this complex quantity. Solution 2 + j1 (2 + j1)(3 - j4) - - = 3 + j4 (3 + j4)(3 - j4) 2 .1 = 5" - '5" Hence, 6+j3-j8+4 9 + 16 10 - j5 25 2 real part = - 5' . . .1 nnagmary part = -,-5 The magnitude and angle of this complex quantity are obtained as follows: magnitude = ~n)' + (~1)' = &= ~ = 0.447 -115 -1 angle = tan-1- - = tan-1- = -26.565° 215 2 Problem A-2-2 Find the Laplace transform of Solution Since f(l) = 0 1

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